⚖ Fort Worth Math Circle - Math Auction ⚖

1. Using the digit 2 five times, represent as many consecutive positive whole numbers as possible, starting from 1. You can use the arithmetic operations $+$, $-$, $\times$, $\div$, and parentheses. The same operations can be used several times, and you are not required to use all four operations.

   - *Example:* $1 = (2 - 2/2) \times 2/2$
   - *Stronger solutions:* A team has a stronger solution for this problem if the team is able to continue the list starting from where the previous team stopped.

2. Using the digits 1, 2, 3, 4, 5, 6 *in this order*, represent as many consecutive positive whole numbers as possible, starting from 1. You can use the arithmetic operations $+$, $-$, $\times$, $\div$, and parentheses. The same operations can be used several times, and you are not required to use all four operations. However, you must use all six numbers and they must appear in increasing order in the final expression.

   - *Stronger solutions:* A team has a stronger solution for this problem if the team is able to continue the list starting from where the previous team stopped.

3. Use seven straight lines to divide a circle into as many triangular pieces as possible. "Triangles" with curved sides don't count, nor do triangles made of smaller triangles or polygons.

   - *Stronger solutions:* A team has a stronger solution for this problem if the team is able to present a drawing that has more triangles.

4. Find as many solutions as possible to the alphanumeric puzzle $\text{BACK} + \text{BOA} = \text{SCAM}$. (Equal letters stand for the same equal digits, and different letters for different digits.)

   - *Stronger solutions:* A team has a stronger solution for this problem if the team can present at least one solution that was not given before.

5. Two circles with different radii are centered at the same point. Pick four distinct points on the outer circle and two distinct points on the inner circle. Connect all the points to get lines. What is the minimal number of straight lines?

   - *Stronger solutions:* A team has a stronger solution for this problem if the team is able to present a solution with fewer lines.

6. Start with the number 123451234512345123451234512345 and cross out ten digits so that the remaining number is as large as possible.

   - *Stronger solutions:* A team has a stronger solution for this problem if the team comes up with a larger number.

7. Take 7 round pizzas, and cut them into wedges so that the 7 pizzas are divided equally among 8 people. Make as few cuts as possible. (A "cut" is a straight line that starts and ends at an outer edge of the pizza and goes through the center of the pizza.) Each of the 8 people should get the same share, and there should be no leftovers. Different people can get a different assortment of slices as long as they add up to equal shares.

   • *Stronger solutions:* A team has a stronger solution for this problem if the team is able to cut the pizzas using fewer cuts.

8. A trader has a gold chain with 20 links. (The chain is not fastened into a loop.) During an expedition, the trader hires a guide for a 20-day trip through the desert. The guide asks him for one gold link per day as payment, and he wants to be paid at the end of each day. In order to make the daily payments, the trader has to open several links of his chain. Since unfastening a link costs money, the trader does not want to open too many links. The good news is that the guide is willing to trade links with the trader to make things even.

   For example, if the trader pays with a single link on day one, he can give the guide a two-link chain on day two and get the single link back.

   What is the smallest number of links the trader would need to open to be able to pay his guide for the trip?

   • *Stronger solutions:* A team has a stronger solution for this problem if the team is able to present a solution with fewer open links.

9. Make the minimum possible number of marks on a wooden plank so that every integer number of inches from 1 through 15 can be measured using this plank; that is, this length can be represented as the distance between some pair of the marks.

   • *Stronger solutions:* A team has a stronger solution for this problem if it has a way using fewer marks.

10. The famous chef, Patty Cake, cooks a cake that has the shape below. This cake is to be cut into four equal parts of exactly the same size and shape. Find as many different ways to cut this cake into four pieces of the same size and shape.
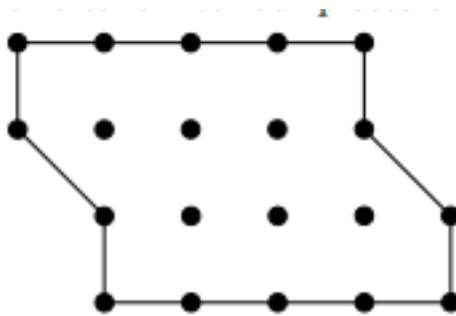
   • *Stronger solutions:* A team has a stronger solution for this problem if the team has solution that has not been presented before.

Rules for Mathematical Auction

1. We divide into teams and work for a fixed amount of time to solve the problems above.

2. Each team is given starting money of $1000 of Math Money.

3. The best solution to a problem is worth $200 Math Money.

4. The problems are put up for auction in the order given. The team with the highest bid is allowed to present their solution.

5. The problem is then put up for bid again (and again) but this time the solution must be better than the previous solution.

6. When no other team wants to buy the problem, the team with the best solution collects the value of the problem. Every team that "bought" the problem pays for their bid, even if they were not the winning solution.

7. If a team can show that they have the best solution (by showing that their solution can not ever be improved) then that team is eligible for $50 Math Money for the problem.

8. Prizes will be doled out at the end of the game based on the Math Money totals.